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DIALETHEISM AND THE IMPOSSIBILITY OF THE WORLD

Ben Martin

This paper first offers a standard modal extension of dialetheic logics that respect the normal semantics for negation and conjunction, in an attempt to adequately model *absolutism*, the thesis that there are true contradictions at metaphysically possible worlds. It is shown, however, that the modal extension has unsavoury consequences for both absolutism and dialetheism. While the logic commits the absolutist to dialetheism, it commits the dialetheist to the impossibility of the actual world. A new modal logic **AV** is then proposed which avoids these unsavoury consequences by invalidating the interdefinability rules for the modal operators with the use of two valuation relations. However, while using **AV** carries no significant cost for the absolutist, the same isn't true for the dialetheist. Although using **AV** allows her to avoid the consequence that the actual world is an impossible world, it does so only on the condition that the dialetheist admits that she cannot give a dialetheic solution to all self-referential semantic paradoxes. Thus, unless there are any further available modal logics that don't commit her to the impossibility of the actual world, the dialetheist is faced with a dilemma. Either admit that the actual world is an impossible world, or admit that her research programme cannot give a comprehensive solution to the self-referential paradoxes.

Keywords: dialetheism, impossible worlds, paraconsistent logics, contradictions.

Dialetheism is the view that there are true contradictions at the *actual* world [Priest 2014: xxiii]. Call the view that there are true contradictions at a possible world 'absolutism'. Intuitively, the entailment relation between the two positions is asymmetrical. Dialetheism entails absolutism, but absolutism doesn't entail dialetheism. This is just a particular case of the general rule that actuality entails possibility but possibility fails to entail actuality. Unsurprisingly, then, the two positions are distinguished in the dialetheic literature (see Beall [2004: 6]). This paper asks whether there is an available logic that adequately models absolutism as a philosophical position distinct from dialetheism while respecting the normal semantics for conjunction and negation.[1]

1. Absolutism

Absolutism isn't equivalent to paraconsistency. Absolutism proposes that a contradiction C is true at a metaphysically possible world, whereas a logic

[1]That is, **Conjunction:** $v(A \wedge B) = \min\{v(A), v(B)\}$; and **Negation:** $v(\sim A) = 1 - v(A)$.

needs only to invalidate explosion, $\{A, \sim A\} \vdash B$, to be a paraconsistent logic.[2] Assuming that contradictions are formalized as '$A \wedge \sim A$', absolutism requires a special type of paraconsistent logic that not only invalidates both the unconjoined $\{A, \sim A\} \vdash B$ and the conjoined $\{A \wedge \sim A\} \vdash B$ forms of explosion, thereby blocking triviality, but also allows contradictions to be assigned the truth-value true. We will call these special paraconsistent logics *dialetheic* logics, as they allow contradictions to be assigned the truth-value true.

Not all paraconsistent logics are dialetheic logics. Both the preservationist logics of Jennings and Schotch [1984] and Brown [2001], and the discursive logic of Jaśkowski [1969], fail to allow for contradictions to be assigned the truth-value true while invalidating explosion. Thus, the set of dialetheic logics is a proper subset of the set of paraconsistent logics, and one can be a paraconsistent logician without being an absolutist, although, on pain of triviality, the inverse isn't true.

However, while there are logics, such as the preservationist and discursive logics, which demonstrate that a paraconsistent logician *isn't* committed to absolutism or dialetheism, no logic yet has been produced which establishes that absolutism doesn't entail dialetheism. Therefore, although we have three apparently conceptually distinct positions,

Dialetheism: There are true contradictions at the actual world
Absolutism: There are true contradictions at a metaphysically possible world
Paraconsistency: Explosion is invalid,

we don't yet have logical evidence for the non-equivalence of absolutism and dialetheism.

Why, though, should we bother with constructing a logic to accommodate absolutism? Well, even if we possess no good reasons at present to believe that there are true contradictions at non-actual possible worlds but not at the actual world, absolutism and dialetheism seem conceptually distinct positions. We should then be able to reflect this conceptual distinction logically by constructing a system that adequately models the absolutist's theory without absolutism's entailing dialetheism. If we find good reason to believe that attempts to achieve this will fail, then we will have an interesting case of possibility entailing actuality. Additionally, our present lack of good reasons for admitting the truth of contradictions at a non-actual possible world without also admitting the truth of contradictions at the actual world doesn't ensure that we won't find such reasons in the near future.[3]

[2]Actually, there are two respects in which the situation is more complex than this. Firstly, there are at least two different forms of explosion, C-explosion $\{A, \sim A\} \vdash B$ and F-explosion $\bot \vdash B$, and there are possible logics in which C-explosion is invalid but F-explosion is valid (see Marcos [2005: ch. 1]). We are concerned solely with C-explosion here, and thus will allow ourselves to refer uniquely to it with the term 'explosion'. Secondly, there are logics such as Johansson's [1937] 'minimal calculus', a positive fragment of intuitionistic logic, in which although $\{A, \sim A\} \vdash B$ cannot be proven, a special instance of explosion such as $\{A, \sim A\} \vdash \sim B$ can. This is not ideal. In principle, one wants a definition of paraconsistency that ensures for any $-$nary placed connective $\bigcirc$ and any arbitrary formulae B and C, both $\{A, \sim A\} \nvdash \bigcirc B$ and $\{A, \sim A\} \nvdash B \bigcirc C$. For more on how this might be achieved, see Urbas [1990]. Although a more fine-grained definition for paraconsistency is required, these necessary alterations would make no difference to our conclusions here. Thus, we can simply use 'those logics which invalidate explosion' as our definition of paraconsistency.

[3]Although we won't speculate on any particular cases here, such reasons might include admitting the possibility of the veridical perception of contradictory states, as suggested in Priest [1999].

Given that it is only in the last 27 years, with Priest's [1987] dialetheic solution to certain semantic self-referential paradoxes, that the truth of contradictions has become philosophically respectable, if contentious, it's no surprise that there's been a lack of effort exerted in the search for such potential cases. We should be willing to speculate philosophically and to attempt to build a logic that accommodates absolutism, even before we have good reason to endorse such a theory, just as Asenjo [1966] built a logic to model true contradictions before the truth of contradictions had philosophical support.

Given that absolutism is a modal thesis, proposing that some contradictions are true at a *possible* world, the position will require a modal extension of a dialetheic logic if it's to distinguish itself from dialetheism. In attempting to construct a logic suitable to model absolutism we will, firstly, propose a modal extension of dialetheic propositional logics respecting the normal semantics for conjunction and negation that provides a standard semantics for the modal operators. Having shown that the logics resulting from these standard semantics have unsavoury consequences for both absolutism and dialetheism, we will then, secondly, propose a new modal logic **AV** that avoids these consequences.

2. A Standard Modal Semantics

In building a modal dialetheic logic with standard modal semantics, for dialectical purposes we will use Priest's [1979] *Logic of Paradox* (**LP**) as our dialetheic propositional logic. However, the same conclusions follow from any zero-order dialetheic logic that has the normal semantics for conjunction and negation. If there are dialetheic logics that fail to fulfil these conditions, such as da Costa's [1974] **C-Systems**, then we need to question the suitability of these logics to model absolutism independently of this paper's considerations.[4]

An interpretation for our modal extension of **LP** is a quadruple $<W$, w_a, R, $\varepsilon>$. W is a set of possible worlds, w_a is a distinguished member of our domain known as the *actual world*, R is a binary relation between sets of worlds known as the *accessibility relation*, and ε is a valuation relation assigning truth-values to world-indexed propositions (or, to put the point another way, to proposition-world pairs).[5] In this logic, valuations are relations between the world-indexed propositions and the set of truth-values $\{1, 0\}$, with each world-indexed proposition taking *at least* one truth-value. As in **LP**, there are no truth-value gaps in the logic. Thus, world-indexed propositions can be *true*, *false*, or *both* (true and false).

The truth-values of complex world-indexed propositions are then defined so as to mirror those of **LP** ($w \in W$):

[4]Dialetheists have so far been hesitant to use dialetheic logics with non-normal semantics for either conjunction or negation. For example, Priest and Routley [1989: 165–6] have criticized the **C-Systems**' negation for being a sub-contrary, rather than a contradictory-forming, operator. Whether all absolutists would exhibit the same hesitancy is an open question.

[5]To ensure the logic's valuation *relation* isn't mistaken for a valuation *function* we are using epsilon here, rather than the customary 'v', to symbolize valuations.

$(A \wedge B)_w \varepsilon 1$ iff $A_w \varepsilon 1$ and $B_w \varepsilon 1$
$(A \wedge B)_w \varepsilon 0$ iff $A_w \varepsilon 0$ or $B_w \varepsilon 0$

$(A \vee B)_w \varepsilon 1$ iff $A_w \varepsilon 1$ or $B_w \varepsilon 1$
$(A \vee B)_w \varepsilon 0$ iff $A_w \varepsilon 0$ and $B_w \varepsilon 0$

$(\sim A)_w \varepsilon 1$ iff $A_w \varepsilon 0$
$(\sim A)_w \varepsilon 0$ iff $A_w \varepsilon 1$

To provide the semantics of the modal operators for our modal extension of **LP,** we need to assume a certain accessibility relation *R*, our binary relation between sets of worlds. However, we don't want to take a stand on which accessibility relation is the most plausible or most suited to the absolutist's philosophical needs here. Therefore, we will assume an arbitrary accessibility relation *R*. The cogency of our point will hold, whichever accessibility relation we use, as long as *some* possible but non-actual worlds are accessible from the actual world. Now, translating the standard semantics for necessity and possibility into our talk of valuations as relations between proposition-world pairs and the set of truth-values {1, 0}, we get:

$(\Box A)_w \varepsilon 1$ iff, for all $w' \in W$ such that wRw', $A_{w'} \varepsilon 1$
$(\Box A)_w \varepsilon 0$ iff, for some $w' \in W$ such that wRw', $A_{w'} \varepsilon 0$
$(\Diamond A)_w \varepsilon 1$ iff, for some $w' \in W$ such that wRw', $A_{w'} \varepsilon 1$
$(\Diamond A)_w \varepsilon 0$ iff, for all $w' \in W$ such that wRw', $A_{w'} \varepsilon 0$

These definitions retain the intuition that a proposition *p* is necessary at a possible world *w* if and only if *p* is true at every world accessible from *w*, that *p* is not necessary at *w* if and only if *p* is false at some world(s) accessible from *w*, that *p* is possible at *w* if and only if *p* is true at some world(s) accessible from *w*, and that *p* is not possible at *w* if and only if *p* is false at every world accessible from *w*. Here then we have a modal extension of **LP** that contains the standard semantics for the possibility and necessity operators.

Now, the modal dialetheic logic that these standard modal semantics produce can be shown to have two problematic consequences, one solely for absolutism and the other for absolutism and dialetheism alike. These consequences motivate a revision of the logic for both positions' sakes.

3. Consequence One: From Possibility to Actuality

Unfortunately for absolutism, it can be shown that the modal dialetheic logic resulting from these standard modal semantics allows contradictions at a possible world to permeate into the modal level. This entails that if a contradiction is true at a possible world accessible from the actual world then there is a true contradiction at the actual world.

Consider a possible world w_1 accessible from the actual world w_a. Following the absolutist's thesis, allow for there to be a proposition *p* at w_1 that takes both truth-values. Given the meaning of conjunction and negation above, $p \wedge {\sim}p$ takes the truth-value true at w_1, as well as taking the truth-value false. There is nothing new here. In **LP**, any proposition q that is the conjunction of a proposition p and p's negation, when p takes both truth-values, also has both truth-values. We, therefore, have a world accessible from w_a at which a contradiction is true. Irrespective of the truth of any contradiction at any other possible world $w \in W$, we have, at the actual world w_a,

$$\Diamond(p \wedge {\sim}p)_{wa} \varepsilon 1.$$

This isn't the end of the story, though. For at *all* possible worlds, either accessible or inaccessible from w_a, it's easy to see that, for every proposition p, the conjunction of p and its negation ${\sim}p$ takes the truth-value false, whatever truth-value p takes, even if $p \wedge {\sim}p$ sometimes also takes the truth-value true. Therefore, given the semantics of the possibility operator above, it's also going to be false at the actual world w_a that $\Diamond(p \wedge {\sim}p)$. This entails, given the meaning of negation above, that at the actual world w_a,

$${\sim}\Diamond(p \wedge {\sim}p)_{wa} \varepsilon 1.$$

Consequently, we can derive a contradiction at the modal level, as the rule of adjunction is valid.[6]

Thus, if the absolutist were to model her theory with this modal dialetheic logic using the standard semantics for the modal operators, by allowing for at least one proposition p to be both true and false at a possible world accessible from w_a, she would be committed to a contradiction at the actual world. Absolutism would become a subspecies of dialetheism.[7] Consequently, this modal extension of dialetheic logics with standard modal semantics fails to ensure the separation of absolutism and dialetheism. If absolutism is to be a distinct philosophical position from dialetheism, then it requires a different modal logic. In particular, the position must use either non-normal semantics for negation and/or conjunction, or non-standard semantics for the modal operators.

In contrast, this consequence of the standard modal semantics is encouraging for the dialetheist. It gives her a new avenue in which to establish that there are some true contradictions at the actual world. Rather than relying on semantic or set-theoretic paradoxes, she can conclude that there are true contradictions at the actual world *if* she can establish that there are true contradictions at non-actual possible worlds accessible from the actual world.

[6]The same point could also be demonstrated here with the necessity operator, given the interdefinability of the modal operators.

[7]Subsequent to the writing of this paper, it was pointed out to me that in Asmus [2012] a similar point is made using model theory. There is one, not inconsequential, difference between our results, however. While Asmus is interested in showing that paraconsistent logicians are committed to dialetheism, given certain assumptions, my interest here is with the non-equivalent task of showing that a standard modal extension of dialetheic logics respecting the normal semantics for conjunction and negation commits an *absolutist* to dialetheism. While the preservationist logics of Jennings and Schotch [1984] and Brown [2001] seem to be counterexamples to Asmus's claim, in that these paraconsistent logics don't entail true contradictions even if we have an interpretational understanding of cases, they aren't relevant to my claim here as they are *not* dialetheic logics.

She can potentially justify her claim that there are true contradictions by looking to the fruitful grounds of possibility. Unfortunately for the dialetheist, the second consequence of the semantics is far less encouraging.

4. Consequence Two: The Possibility of Impossibility

Interpreted naturally, a consequence of the modal extension of **LP** above is that the absolutist is committed to saying that it's *im*possible for contradictions to be true. By necessitation of the theorem $\sim(p \wedge \sim p)$ we can derive $\Box\sim(p \wedge \sim p)$, and given the interdefinability of the necessity and possibility operators we can derive $\sim\Diamond(p \wedge \sim p)$. Now, interpreted naturally, this formula reads as 'It's impossible for the conjunction of a proposition *p* and *p*'s negation to be true', or 'It's impossible for a contradiction to be true.' Given that an impossible world is a world *w* where propositions that cannot possibly be true *are true*, any world *w* at which a contradiction is true is going to be an impossible world, according to the modal semantics given above. In conjunction with the absolutist's hypothesis that there are possible worlds at which contradictions are true, this entails that the absolutist is committed to the proposition that at least some *impossible* worlds are *possible* worlds. Yet to admit that some impossible worlds are also possible worlds seems to strip impossibility of the theoretical role that the concept plays. If a world *w*'s being an impossible world doesn't preclude that it's also a possible world, then it's unclear what function the concept of impossibility serves. Consequently, by endorsing this modal dialetheic logic with standard modal semantics, the absolutist would be taking on the burden of explaining what theoretical role the concept of impossibility plays if it doesn't preclude possibility.[8]

Additionally, by allowing possibility and impossibility to intersect, this modal dialetheic logic also entails the troubling consequence that absolutism cannot logically preclude the actual world being an *impossible* world. Imagine that the absolutist accepts the modal dialetheic logic above and then we find good reason to believe that *at the actual world* there is a true contradiction, which isn't precluded by the absolutist's theory. This would commit the absolutist to the thesis that the actual world, as well as being a possible world, is an impossible world, given that a contradiction would be true at it. This is somewhat perplexing. After all, an impossible world is one that couldn't be realized, whereas the actual world is. Whatever the actual world is, it doesn't seem to be an impossible world. However, by using the standard modal semantics above, and not precluding the truth of contradictions at the actual world, the absolutist would fail to preclude the impossibility of the actual world, again placing a considerable burden on her theory. Thus, by using this modal dialetheic logic, the absolutist would take on the burden of

[8]As this unsavoury consequence of the standard modal semantics depends upon the notion of impossible worlds, any argument based upon this consequence must resist any concerns over the coherence of impossible worlds, such as found in Lewis [1986]. Although any suitable response to these concerns is far beyond the scope of this paper, the theoretical usefulness of impossible worlds, as argued for by Nolan [1997], suggests that we may well be able to speak coherently about them. Many thanks to an anonymous referee for pointing out this concern.

providing an account of impossibility that made it plausible both for some possible worlds to be impossible worlds and for the concept of impossibility to fail to preclude the impossibility of the actual world.

This unsavoury consequence of the modal dialetheic logic above is equally, if not more, troublesome for the dialetheist. The dialetheist, like all of us, needs to be able to model modal claims, and given that she allows for true contradictions at the *actual* world, she needs to allow for true contradictions at *possible* worlds. The most obvious way of her achieving this, however, is by providing a modal dialetheic logic with the standard modal semantics. Yet a consequence of this logic is that all worlds at which contradictions are true are impossible worlds. Given that the dialetheist endorses the truth of some contradictions at the actual world, the logic subsequently commits her to the actual world being an impossible world. This, again, seems to put strain on the whole notion of an impossible world. Consequently, by using the modal dialetheic logic with standard modal semantics above, dialetheism would also take on the burden of providing a plausible definition of impossibility that can accommodate the impossibility of the actual world.

Nothing that has been said here precludes the possibility of the absolutist or dialetheist providing just such a plausible definition of impossibility that accommodates the impossibility of some possible worlds, which includes the actual world in the dialetheist's case. [9] After all, once we admit that contradictions can be true many wonderful things become possible. However, we *can* say with some confidence both that the dialetheist has shown no sign so far of being willing to accept that the actual world is an impossible world,[10] and that it isn't obvious that the definitions of impossible worlds and situations that prominent dialetheists use can plausibly accommodate the impossibility of the actual world. For example, Priest [2014: xxiii] defines an impossible world as 'one where the laws of logic are different from those of the actual world', while Beall defines impossible situations as situations that can 'never be *actualized*' and 'never [be] part of any possible world' [Beall and Restall 2001: sec. 4]. Now, given that both Priest and Beall are dialetheists, it's an option for either to accept the respective contradictions that would arise from their definitions of impossible worlds or situations if they admitted that the actual world was both a possible and impossible world.[11] While Priest could accept that the logical laws of the actual world are both identical and non-identical to themselves, Beall could propose that impossible worlds can be both actualized and not actualized. However, dialetheists don't wish to accept the truth of just any contradiction. As with all propositions, they only wish to endorse those contradictions that we have good reason to believe are true. Therefore, if the dialetheist wants to endorse either of these contradictions, we are owed an explanation for both why we have

[9] Many thanks to an anonymous referee for pressing me on this point.

[10] Priest makes it clear in an added footnote in Lewis [2004] that he doesn't want to say that true contradictions at the actual world entail that the actual world is an impossible world.

[11] Beall's definition of impossible *situations* only entails a contradiction in conjunction with the impossibility of the actual world if we assume that impossible *worlds* contain impossible *situations*, so that if an impossible situation can never be actualized then neither can an impossible world. However, this seems a reasonable assumption to make for the sake of our discussion.

good reason to believe they are true and why they are theoretically unproblematic for the concept of impossibility. At present, not enough has been said in the literature on the repercussions of dialetheism for our common understanding of impossibility and impossible worlds.

The absolutist and dialetheist have the choice, therefore, of either (a) or (b):

(a) endorsing a modal dialetheic logic that both respects the normal semantics for negation and conjunction and uses standard modal semantics, such as the logic given above, whilst taking on the burden of accommodating worlds that are both possible and impossible and, in the dialetheist's case, the impossibility of the actual world;

(b) avoiding these consequences by endorsing a different modal dialetheic logic, which requires using either non-normal semantics for negation and/or conjunction or non-standard semantics for the modal operators.

Consequently, to avoid either of the unsavoury consequences we have considered, an alternative modal dialetheic logic is required. If no suitable alternative logic can be found then, firstly, absolutism will be condemned to the status of being a sub-species of dialetheism. Secondly, the absolutist will be required to offer an explanation of how we can make sense of the impossibility of some possible worlds, and the dialetheist will be committed to the actual world being an impossible world. Thus, if there are no available logics that avoid either of these unsavoury consequences, then both absolutism and dialetheism face substantial philosophical challenges.

5. Possible Solution to the Problem

If the absolutist is going to block both the occurrence of true contradictions at the actual world and the possibility of impossible worlds, then she will need to invalidate the interdefinability rules for the modal operators. While, through necessitation, $\Box{\sim}(p \wedge {\sim}p)$ must be a theorem of the absolutist's logic, given the normal semantics for conjunction and negation, and she must be able to derive $\Diamond(p \wedge {\sim}p)$ to allow for true contradictions at possible worlds, these commitments cause problems when conjoined with the interdefinability of the modal operators. As the absolutist cannot sanction the rejection of either commitment, the interdefinability of the modal operators must be invalidated somehow. While the occurrence of true contradictions at the actual world is caused by both interdefinability rules,

$$\Diamond A = {\sim}\Box{\sim}A$$
$$\Box{\sim}A = {\sim}\Diamond A,$$

the second consequence of the standard modal semantics above, that some impossible worlds are possible worlds, is a consequence of the second interdefinability rule. Thus, the absolutist can avoid both of the unsavoury consequences by invalidating the interdefinability rules for the modal operators.

However, given that the absolutist is committed to both allowing for true contradictions and, under the present proposal, respecting the normal semantics for negation and conjunction, she cannot invalidate the interdefinability rules for the modal operators by just any means. Firstly, if negation is to keep its truth-reversing properties, as dictated by its normal semantics, then the only way to block the occurrence of contradictions involving modal formulae, and consequently the intersection of possibility and impossibility, is to ensure that modal formulae never take both truth-values. Consequently, the absolutist must invalidate the interdefinability rules for the modal operators whilst ensuring that modal formulae only take one truth-value. Secondly, given that the absolutist must still allow for true contradictions by ensuring that *non-modal* formulae can be assigned both truth-values, it's clear that she must ensure that modal formulae *cannot* be assigned both truth-values by altering the semantics for the modal operators, rather than by precluding *tout court* the possibility of formulae being assigned both truth-values.

To see how the absolutist could alter her modal semantics to preclude the possibility of modal formulae being assigned both truth-values, whilst allowing non-modal propositions to be both true and false, we can look to how one could intuitively avoid the intersection of possibility and impossibility whilst assuming a dialetheic propositional logic. Given the normal semantics of negation, formulae of the form $\sim\Diamond A$ take the truth-value true at a world w if and only if $\Diamond A$ is false at w. Consequently, given standard modal semantics, $\sim\Diamond A$ is true at w in virtue of A being false at all worlds accessible from w, which means that A is impossible at w in virtue of being false at the worlds accessible from w. Given a dialetheic propositional logic in which propositional parameters can be assigned both truth-values, however, these modal semantics don't preclude both $\Diamond A$ and $\sim\Diamond A$ being true at w, as A may take both truth-values at some accessible world. Thus, to ensure that $\Diamond A$ and $\sim\Diamond A$ cannot both be true at a world w in such a logic, one needs to provide the conditions under which $\Diamond A$ is false at a world, and thus the conditions under which $\sim\Diamond A$ is true, not in terms of the *falsity* of A at accessible worlds, which doesn't preclude A's truth at these worlds, but in terms of another property, such as A's *failing to be true* at these accessible worlds. By providing the truth conditions of the modal operators in terms of a proposition's being true at the relevant accessible worlds, and the falsity conditions in terms of a proposition's *failing to be true* at the relevant accessible worlds, these semantics would ensure that modal formulae couldn't take both truth-values, provided we could successfully preclude the possibility of a proposition both being true and *failing to be true*. If realized, these semantics would allow for the absolutist to retain the standard conditions under which A is possible at a world w, whilst ensuring the mutual exclusivity of possibility and impossibility by changing the conditions under which A is *impossible* at w (and similarly for necessity).

To successfully preclude the possibility of modal formulae being assigned both truth-values, this solution requires the absolutist to possess the logical apparatus necessary to ensure that the truth-value true *isn't* a member of the truth-values that a proposition-world pair takes. This we will achieve by proposing a new modal logic that posits two primitive valuation relations, rather than one.

Our new logic **AV** is a quintuple, $<W, w_a, R, \varepsilon^+, \varepsilon^->$. Only the final element differs from our previous logic, with 'ε^+' symbolizing the valuation relation for the logic, just as 'ε' did before. This new element, ε^-, rather than symbolizing the *valuation* relation for the logic, symbolizes the logic's *anti-valuation* relation. Although the idea of having two valuation relations in a logic may seem bizarre, it's worth persevering with as its results are fruitful for both the absolutist and dialetheist. There is an obvious analogy at work here between the valuation and anti-valuation relations and the extension and anti-extension of a predicate. Thus, a proposition-world pair p_w will have both a *valuation* set and an *anti-valuation* set. The *valuation* set of p_w will be dictated by the truth-values to which p_w has the relation ε^+, and the *anti-valuation* set of p_w will be dictated by the truth-values to which p_w has the relation ε^-. We will presently discuss some of the properties that the two valuation relations possess. With regards to the accessibility relation R, again we don't want to make too many assumptions about the accessibility relation which would best suit the absolutist and dialetheist. All we require for our purposes here are the weak requirements that the relation is reflexive and that the distinguished world w_a has some non-distinguished possible worlds accessible from it.

As before, valuations are relations from proposition-world pairs to the set of truth-values $\{1, 0\}$, and our anti-valuations here are similarly relations from proposition-world pairs to the set of truth-values $\{1, 0\}$. Now, for the semantics to deliver the results we require, we must make two assumptions about the valuation and anti-valuation sets for each proposition-world pair. Firstly, we need to assume that the valuation and anti-valuation sets partition the set of truth-values {true, false} for each proposition-world pair. Thus, for any proposition-world p_w and truth-value t:

Either $p_w\varepsilon^+t$ or $p_w\varepsilon^-t$, and it's not the case that both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$.

These conditions ensure that for every proposition-world pair p_w, each truth-value t is a member of p_w's *valuation* or *anti-valuation* set, but not both.[12] The second assumption, to ensure that the semantics for the logic aren't gappy, is that the *valuation* set for every proposition-world pair p_w must be non-empty. These assumptions ensure that although a proposition-world pair p_w can have the valuation relation to both true and false, p_w can only have the *anti*-valuation relation to either true or false.

Given these restrictions on the anti-valuation relation, it seems reasonable to consider the relation to be communicating which truth-values are *not* members of the valuation set of a proposition-world pair p_w. Thus, the anti-valuation set of a proposition-world pair p_w is hypothesized, at least, to be able to communicate when p_w is *un*true ($p_w\varepsilon^+0$ and $p_w\varepsilon^-1$) and *un*false ($p_w\varepsilon^+1$

[12]One might rightly wonder whether we can simply stipulate that a truth-value t cannot be a member of both the valuation and anti-valuation set of a proposition-world pair p_w. After all, the possibility of such stipulated mutual exclusivity failing is one of the lessons learnt from dialetheism. We will consider this possibility later.

and $p_w\varepsilon^-0$), as well as both true and false ($p_w\varepsilon^+\{1,0\}$ and $p_w\varepsilon^-\emptyset$). This makes the anti-valuation relation similar to the classical negation used when claiming that a proposition p is '**not** true' or '**not** false', with the intention of precluding p's falsity and truth, respectively. Priest [1990, 2007: 469–71] has argued that we cannot interpret these **not**s classically without begging the question against the dialetheist. **AV** makes this hypothesized mutual exclusivity of the relations explicit by stipulating that for no proposition-world pair p_w and truth-value t is it the case that both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. Again, whether we can ensure such mutual exclusivity through stipulation is something we will discuss later. The hope for **AV** is that it can avoid accusations of begging the question against the dialetheist, as the dialetheist's endorsement of the logic is ultimately in her best interests.

Critically for the absolutist and dialetheist, **AV**'s semantics allow for contradictions to be true at a world. We can see this by giving the dual truth-conditions of the truth-functional connectives using both valuation relations:

$(A \wedge B)_w\varepsilon^+1$ iff $A_w\varepsilon^+1$ and $B_w\varepsilon^+1$
$(A \wedge B)_w\varepsilon^+0$ iff $A_w\varepsilon^+0$ or $B_w\varepsilon^+0$
$(A \wedge B)_w\varepsilon^-1$ iff $A_w\varepsilon^-1$ or $B_w\varepsilon^-1$
$(A \wedge B)_w\varepsilon^-0$ iff $A_w\varepsilon^-0$ and $B_w\varepsilon^-0$

$(A \vee B)_w\varepsilon^+1$ iff $A_w\varepsilon^+1$ or $B_w\varepsilon^+1$
$(A \vee B)_w\varepsilon^+0$ iff $A_w\varepsilon^+0$ and $B_w\varepsilon^+0$
$(A \vee B)_w\varepsilon^-1$ iff $A_w\varepsilon^-1$ and $B_w\varepsilon^-1$
$(A \vee B)_w\varepsilon^-0$ iff $A_w\varepsilon^-0$ or $B_w\varepsilon^-0$

$(\sim A)_w\varepsilon^+1$ iff $A_w\varepsilon^+0$
$(\sim A)_w\varepsilon^+0$ iff $A_w\varepsilon^+1$
$(\sim A)_w\varepsilon^-1$ iff $A_w\varepsilon^-0$
$(\sim A)_w\varepsilon^-0$ iff $A_w\varepsilon^-1$

If a proposition-world pair p_w has the valuation relation to both truth and falsity, $p_w\varepsilon^+\{1,0\}$, then both $p_w\varepsilon^+1$ and $\sim p_w\varepsilon^+1$. Consequently, we have $(p \wedge \sim p)_w\varepsilon^+1$, which ensures that we can have true contradictions at a world, while retaining the intuitive consequence that all contradictions are false at every world, $(p \wedge \sim p)_w\varepsilon^+0$. **AV** therefore fulfils the suitability requirement for the absolutist and dialetheist of allowing contradictions to be assigned the truth-value true.

What we now need are semantics for the modal operators that invalidate the interdefinability rules. We can achieve this by giving the semantics for the modal operators exclusively in terms of truth:

$(\Box A)_w\varepsilon^+1$ iff, for all $w' \in W$ such that wRw', $A_{w'}\varepsilon^+1$
$(\Box A)_w\varepsilon^+0$ iff, for some $w' \in W$ such that wRw', $A_{w'}\varepsilon^-1$
$(\Diamond A)_w\varepsilon^+1$ iff, for some $w' \in W$ such that wRw', $A_{w'}\varepsilon^+1$
$(\Diamond A)_w\varepsilon^+0$ iff, for all $w' \in W$ such that wRw', $A_{w'}\varepsilon^-1$

We don't require the dual anti-valuation conditions for the modal operators, as they are redundant for our purposes. This is ensured by **AV**'s not permitting a truth-value t to be a member of both the valuation and anti-valuation sets of a proposition-world pair p_w.

Whether modal formulae have the valuation true or false is dependent only on whether the formulae in question have the valuation or anti-valuation relation to true at the relevant possible worlds. Under the assumption that for no world-proposition pair p_w and truth-value t is it the case that both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$, the truth and falsity of modal propositions become mutually exclusive, invalidating the interdefinability rules for the modal operators and, consequently, blocking the occurrence modal contradictions.

While the formula $\Diamond(p \wedge {\sim}p)$ takes the valuation *true* at the actual world w_a in some interpretations, as the absolutist allows for a possible world w' accessible from w_a such that $(p \wedge {\sim}p)_{w'}\varepsilon^+1$, the formula ${\sim}\Box{\sim}(p \wedge {\sim}p)$ only takes the valuation *false* at w_a in every interpretation, as $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^+1$ but *not* $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^+0$. Given that, for every possible world w accessible from w_a, ${\sim}(p \wedge {\sim}p)_w\varepsilon^+1$, the occurrence of a possible world w' accessible from w_a such that ${\sim}(p \wedge {\sim}p)_{w'}\varepsilon^-1$ is precluded. Therefore, given **AV**'s semantics for the necessity operator and negation, we don't have $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^+0$, or consequently ${\sim}\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^+1$, in any interpretation that $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+1$. The interdefinability of $\Diamond A$ and ${\sim}\Box{\sim}A$ fails.

Similarly, while $\Box{\sim}(p \wedge {\sim}p)$ has the valuation *true* at w_a in every interpretation, for at every world w accessible from w_a ${\sim}(p \wedge {\sim}p)_w\varepsilon^+1$, ${\sim}\Diamond(p \wedge {\sim}p)$ only has the valuation *false* at the actual world w_a in some interpretations. Given that $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+1$ in some interpretations, for the reasons already given, the occurrence of ${\sim}\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+1$ is precluded in those interpretations as this would require both $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+1$ and $\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+0$, which subsequently requires there to be a world w' accessible from w_a at which both $(p \wedge {\sim}p)_{w'}\varepsilon^+1$ and $(p \wedge {\sim}p)_{w'}\varepsilon^-1$, which is precluded by **AV**'s semantics. Therefore, there are interpretations of **AV** in which we don't have both $\Box{\sim}(p \wedge {\sim}p)_{wa}\varepsilon^+1$ and ${\sim}\Diamond(p \wedge {\sim}p)_{wa}\varepsilon^+1$. The interdefinability of $\Box{\sim}A$ and ${\sim}\Diamond A$ fails.

As we have stipulated that a truth-value t cannot be a member of both a proposition-world pair p_w's valuation and anti-valuation sets, we can easily show that there's no interpretation in which either $(\Box A \wedge {\sim}\Box A)_w\varepsilon^+1$ or $(\Diamond A \wedge {\sim}\Diamond A)_w\varepsilon^+1$, for any formula A and world w. Given the truth-conditions of conjunction and negation above, for the modal contradictions to have the valuation true would require that $(\Box A)_w\varepsilon^+\{1,0\}$ and $(\Diamond A)_w\varepsilon^+\{1,0\}$, respectively. However, for either to occur in an interpretation would require that A contained the truth-value true in both its valuation and anti-valuation sets at some world w' accessible from w. Given that both $A_{w'}\varepsilon^+1$ and $A_{w'}\varepsilon^-1$ can't occur together in an interpretation in **AV**, no contradictions constituted by modal formulae have the *valuation* true in **AV**. That there is no interpretation in which $(\Diamond A \wedge {\sim}\Diamond A)_w\varepsilon^+1$ is also enough to demonstrate that there's no world w that is both a possible and impossible world. Possibility and impossibility cannot intersect in **AV**. So, if for some contradiction C we have $C_{wa}\varepsilon^+1$, as the dialetheist theorizes, we are *not* then committed to the actual world being an impossible world, as **AV**'s semantics don't permit ${\sim}\Diamond C_{wa}\varepsilon^+1$, given that $\Diamond C_{wa}\varepsilon^+1$ is ensured by R's reflexivity. Accordingly, given **AV**'s consequence relation,

$\Sigma \models_{AV} B$ iff for all interpretations $<W, w_a, R, \varepsilon^+, \varepsilon^->$ and all $w \in W$, if $A\varepsilon^+1$ for all $A \in \Sigma$, then $B\varepsilon^+1$,

propositions of the form $\sim\Diamond(A \wedge \sim A)_w$ aren't logical truths in **AV**, as in some interpretations there's a possible world w' such that $\Diamond(p \wedge \sim p)_{w'}\varepsilon^+1$.[13]

AV delivers an intuitive semantics for the modal operators whilst ensuring that both of the interdefinability rules are invalidated. Consequently, both of the unsavoury consequences of the modal dialetheic logic with standard modal semantics we considered can be avoided. **AV** delivers everything we need from it, on the assumption, at least, that for no truth-value t and proposition-world pair p_w is there a permitted interpretation in which both $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. It is to this stipulated partitioning of the set of truth-values that we now move, along with the dilemma it poses for the dialetheist.

6. Stipulating Exclusivity and a Dilemma for the Dialetheist

The exclusivity of truth and falsity for modal formulae, and thus modal propositions, is ensured only by the hypothesized mutual exclusivity of the valuation and anti-valuation relations ε^+ and ε^- for every proposition-world pair p_w and truth-value t. If this mutual exclusivity breaks down then so does the mutual exclusivity of truth and falsity for modal formulae, and the same problems reappear for absolutism. Thus, we need assurances that the absolutist can guarantee that $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$ cannot occur together in an interpretation for any proposition-world pair p_w and true-value t. Yet we know that simply stipulating mutual exclusivity isn't enough. Just as self-referential sentences threaten the mutual exclusivity of truth and falsity although the classical logician stipulates their mutual exclusivity, so self-referential sentences can threaten the mutual exclusivity of $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. Consequently, if the absolutist is going to make **AV**'s semantics viable, and ensure that her position doesn't have the previously mentioned unsavoury consequences, she faces the same challenge that the classical logician does—to give a non-dialetheic response to certain self-referential sentences. If the absolutist wishes to stop her position from dissolving into dialetheism, however, this is hardly a new obligation. Allowing for true contradictions at non-actual possible worlds, whilst demurring on the question of true contradictions at the actual world, already required her to avoid a dialetheic solution to the self-referential paradoxes. If the mutual exclusivity stipulated by **AV** requires something more of the absolutist who doesn't want to endorse dialetheism, it's simply that she must double her efforts to give non-dialetheic solutions to the self-referential paradoxes. Otherwise, absolutism is destined to be a sub-species of dialetheism.

The predicament for the dialetheist with regard to **AV** is far more interesting. The dialetheist herself requires the semantics of **AV** if she's to block the

[13] **AV** possesses further interesting properties whose details are beyond the scope of this paper. These include **AV**'s validation of two forms of modal explosion, $\{\Box A \wedge \sim\Box A\} \models_{AV} B$ and $\{\Diamond A \wedge \sim\Diamond A\} \models_{AV} B$, and the potential inclusion of a consistency operator in **AV** that allows for the recapture of classical validity (as in da Costa's [1974] **C-Systems**).

unsavoury consequence that the actual world is an impossible world. For **AV** to achieve this, however, the dialetheist must maintain the mutual exclusivity of $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$ for every proposition-world pair p_w and true-value t. Yet the dialetheist is famously uneasy with such stipulated mutual exclusivity, due to our ability in natural languages to form troublesome self-referential sentences such as (λ):

(λ) λ has the anti-valuation true [$\lambda\varepsilon^-1$].

As with other self-referential sentences, the dialetheist would like to say that (λ) both has and doesn't have the anti-valuation true, entailing that both $\lambda_w\varepsilon^+1$ and $\lambda_w\varepsilon^-1$, contrary to the stipulation of **AV**. This desire to give a dialetheic solution to (λ) poses a dilemma for the dialetheist.

The dialetheist, if she's to avoid taking on a substantial theoretical burden, needs to guarantee that her theory doesn't entail that the actual world is an impossible world. This requires, under the present proposal of **AV**, ensuring the mutual exclusivity of $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$ for all proposition-world pairs p_w and truth-values t. Yet the only way in which the dialetheist can maintain this mutual exclusivity is by giving a non-dialetheic solution to (λ) above. Therefore, unless a new modal semantics can be introduced that removes this dilemma, the dialetheist must choose between endorsing the claim that the actual world is an impossible world or admitting that there is at least one self-referential sentence that cannot be given a dialetheic response, putting at risk the comprehensiveness of her own research programme.[14]

7. Conclusion

In **AV** we have found a modal dialetheic logic that possesses the properties necessary to model absolutism as a philosophical position distinct from dialetheism while respecting the normal semantics for negation and conjunction. However, to ensure **AV**'s viability for her purposes the absolutist must still meet the challenge of self-referential sentences such as (λ) that threaten the mutual exclusivity of the valuation relations. There is nothing we can do *within* **AV** to preclude conclusively the relations' non-exclusivity; this is something that dialetheism has taught us. If the absolutist is to prevent absolutism from dissolving into dialetheism, she must argue for the valuation relations' mutual exclusivity by dealing head on with self-referential sentences such as (λ). Fortunately for the absolutist, unless the dialetheist has any other modal dialetheic logics available to her that don't entail the impossibility of the actual world, the dialetheist herself has good reason not to dispute the stipulated mutual exclusivity of $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. After all, for the dialetheist, it is a choice between accepting **AV**'s semantics and taking on the

[14]This paper leaves open the possibility of there being other modal dialetheic logics that avoid the consequence that the actual world is an impossible world *without* compromising the comprehensiveness of the dialetheist's research programme. If the dialetheist wants to avoid this dilemma, however, then the onus is on her to find such a logic.

burden of accommodating the impossibility of the actual world. Therefore, in **AV**'s facilitating the dialetheist's avoidance of this consequence, the absolutist may find assurances that the dialetheist will be as determined as she is to maintain the mutual exclusivity of $p_w\varepsilon^+t$ and $p_w\varepsilon^-t$. Those who would normally question the mutual exclusivity of semantic categories have, on this occasion, good reason to argue for their exclusivity, though at the risk of damaging the comprehensiveness of their own research programme by admitting a non-dialetheic solution to (λ). Consequently, unless there are alternative modal dialetheic semantics available which don't entail the impossibility of the actual world, the sole possibility threatening **AV**'s suitability to model absolutism is the potential for the dialetheist to bite the bullet and to admit that the actual world is indeed an impossible world.[15]

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